EXODUS COLLEGE SCHOOL WAKISO
S.5 END OF TERM 2 2026
UGANDA ADVANCED CERTIFICATE OF EDUCATION
PRINCIPAL MATHEMATICS 1
PAPER 1
Duration: 3 hours

INSTRUCTIONS TO CANDIDATES:

- The paper consists of four items
- Item 1 is **compulsory** . Choose any more two items of your choice
- Attempt three items in total
- Begin each new item on a fresh page.

Item 1 (Compulsory)

Akira Engineering Ltd in Wakiso has been contracted to design a new sports complex. The project manager, Sarah, a final year mathematics student, is tasked with two designs: a running track and a triangular spectator stand.

For the **running track**, the design team proposed that the center line of the main straight should pass through the point $(5, 3)$ and be perpendicular to the access road whose equation is $2x - y + 4 = 0$.

For the **spectator stand**, the structure will be triangular with corners labeled A, B and C. The engineers need to verify structural balance using trigonometry. They have derived that for any triangle, the relationship between angles and sides must satisfy certain identities.

The lighting consultant also needs to position 2 spotlights. The angle of the first spotlight is modeled by $\cos(\theta + 30°)$ and the second by $\cos(\theta + 48°)$. The difference in illumination must satisfy the equation $\cos(\theta + 30°) - \cos(\theta + 48°) = 0.2$ for optimal coverage.

TASK

(a) Determine the equation of the center line for the running track.

(b) The lighting model needs simplification.

(i) Express $\cos(\theta + 30°) - \cos(\theta + 48°)$ in the form $R \sin P \sin Q$, where R is a constant.

(ii) Hence solve the equation

$$\cos(\theta + 30°) - \cos(\theta + 48°) = 0.2$$

for $0° \leq \theta \leq 360°$.

(c) To prove the structural stability of the triangular stand, show that in any triangle ABC:

$$\frac{\sin(A - B)}{\sin(A + B)} = \frac{a^2 - b^2}{c^2}$$

Item 2

A large-scale poultry farmer is conducting a comprehensive audit of his farm records to optimize operational efficiency and cut unnecessary expenses. He needs to evaluate three core areas of his business: feed procurement costs, automated flock hydration safety, and wholesale egg inventory marketability.

The farm's recent inventory ledger shows three bulk purchases of Layers Mash, Maize Bran, and Protein Concentrate as shown below

	Mass of the feeds × 100 Kgs		
Purchase	Layers mash	Maize brand	Protein concentrate
A	3	1	2
B	1	2	1
C	2	1	3

The total amount used in the purchases A, B and C was 2,800,000, 1,800,000 and 2,600,000/= respectively. The farmer will consider these purchases cost friendly if he is to use less than 5,500,000 to purchase 400 kg of layers, 400kg of maize brand and 300kg of protein concentrate.

The farm uses an automated drinking system where a vital vitamin boost is dissolved into the flock's daily water supply. Sensor telemetry records show that the volume of water consumed during the daytime is $\sqrt{3x+700}$ litres, while the volume consumed at night is $\sqrt{x+600}$ litres. The automated system logs show that the total water volume consumed by the birds over a full 24-hour cycle is exactly 70 litres. The vitamin boost will successfully optimize egg production if its mass x doesn't exceed 300g.

The farm currently has an inventory of 33 trays of eggs sitting in storage. Due to market volatility, the farm's management software implements a strict regulatory rule to determine whether a batch of eggs is cleared for immediate sale or held back. A batch can only be sold if 130 times the number of available trays is strictly greater than the square of the number of trays plus 3000. The farmer does not know whether his current stock meets this safety margin to be cleared for the market.

Task: Help the poultry farmer to make decisions on his farm using knowledge of algebra

Item 3

PrecisionTech Ltd, a manufacturing company in Kampala, produces microchips. The quality control team uses binomial models to estimate defect rates.

For a new production batch, the probability of a chip being defective is modeled as $\frac{1}{x}$. The team wants to analyze the probability that 5 chips sampled are all non-defective.

The finance department also needs to verify a cost-saving formula. The total savings over n months is claimed to follow the pattern:

$$S_n = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Tasks:

(a) The engineers model the non-defective probability using the binomial expansion.

(i) Find the binomial expansion of $\left(1-\frac{1}{x}\right)^5$.

(ii) Use your expansion to estimate $(0.875)^5$ to four decimal places.

(iii) State the range of values for which the binomial expansion is valid.

(b) The research team proposes that the sum of the first n square numbers is given by:

$$1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$

Prove this result by mathematical induction.

Item 4

GreenGrow Agro Ltd, a youth agribusiness in Iganga, produces organic fertilizer. The company is expanding and the HR manager must design a 6-month incentive plan for sales agents. The finance director has set a budget cap of Shs. 12,000,000 per agent for the 6 months.

Two payment schemes are proposed:

- **Scheme A:** An agent earns Shs. 6,000,000 in month 1. Each subsequent month the earning is tripled.
- **Scheme B:** An agent starts with Shs. 300,000 in month 1. Each month the earning increases by a constant amount of Shs. 150,000.

The board will only approve a scheme if the total payout over 6 months does not exceed the budget cap. If both are within budget, the scheme with the higher total payout will be chosen to motivate staff.

The production team models the cost of packaging using the function:

$$C(x)=\frac{11x-1}{(1-x)^2(2+3x)}$$

where x represents hundreds of fertilizer bags produced. They need this expressed in partial fractions to estimate costs for different production levels.

The sales department must export at least 60 tonnes of fertilizer in 12 months to keep their EU license. The operations manager plans to use N distribution trucks. Each truck is designed to deliver $(N-3)$ tonnes per month. The company yard can only accommodate a maximum of 12 trucks.

Tasks:

(a) Justify the board's choice by calculating the total expenditure for each scheme and comparing it against the budget cap of Shs. 12,000,000.

(b) Express the cost function $C(x)$ in partial fractions to help the production team with cost analysis.

(c) Determine the number of distribution trucks, N, the operations manager should deploy to meet the export target without exceeding the yard capacity.